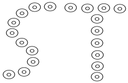

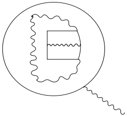

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James Edwards

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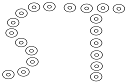

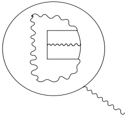

Strings, Wilson Loops and QED

James Edwards

Durham University

Supervised by Prof. Paul Mansfield

Outline

Introduction

There is a long history of association between string theories and gauge theories

1. Flux tubes in QCD
2. Nambu[1] and Polyakov loops[2].
3. Bern-Kosower rules[3].
4. ADS / CFT

[1] Phys Lett B **80**
[2] Nucl Phys B **164**
[3] Arχiv:0101036v2 (Review)

Introduction

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Our work hopes to use string inspired actions to reproduce Maxwell's electromagnetism and a Wilson loop formulation of QED.

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Motivation

The work was initially motivated out of an attempt to explain *Gauss' Law* to undergraduates

$$\int_{\partial V} \mathbf{E} \cdot d\mathbf{s} = \int_V \rho \; dV \tag{1}$$

interpreted in terms of flux lines.

Two (related) problems in interpretation arise:

- What is the physical interpretation of the flux lines?
- How can a discrete counting of individual flux lines reproduce a continuous theory?

Electric Field

In our work we treat the lines of flux as physical objects. We consider statistical averages over fluctuations of the lines with a carefully chosen weight (action)

$$\mathbf{E}(\mathbf{x}) = \int \mathscr{D}\mathbf{y} \int_{\mathbf{a}}^{\mathbf{b}} \delta^3(\mathbf{x}-\mathbf{y})\, d\mathbf{y} \exp\left(-\int_0^T dt \frac{1}{2}\dot{\mathbf{y}}^2\right) \tag{2}$$

In the limit that $T \to \infty$ this average reproduces the electrostatic dipole field[4].

[4] Arχiv:1108.5094v2

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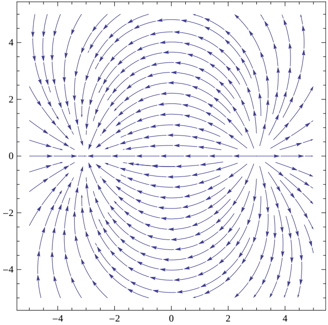

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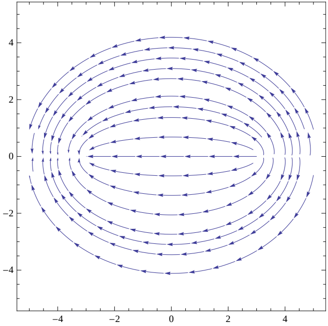
In the limit that $T \to 0$?



Electromagnetism

To move on to covariant electromagnetism we introduce the field strength tensor $F_{\mu\nu}$ and use the equation of motion

$$\partial_\mu F^{\mu\nu} = J^\nu \tag{4}$$

This time we consider fluctuations over surfaces bounded by the world-lines of the moving particles under the Nambu-Goto (statistical) action

$$F_{\mu\nu}(\mathbf{x}) = \int \mathscr{D}\mathbf{y} \int_\Sigma \delta^4\left(\mathbf{x} - \mathbf{y}\right) d\Sigma_{\mu\nu}(\mathbf{y}) \exp\left(\int d^2\xi \sqrt{g}\; g^{ab} \frac{\partial y^\mu}{\partial \xi^a} \frac{\partial y^\nu}{\partial \xi^b} G_{\mu\nu}\right) \tag{5}$$

Time asymmetry

$$F_{\mu\nu}(\mathbf{x}) = \int \mathscr{D}\mathbf{y} \int_{\Sigma} \delta^4 \left(\mathbf{x} - \mathbf{y}\right) d\Sigma_{\mu\nu}(\mathbf{y}) \exp \left(\int d^2\xi \sqrt{g}\ g^{ab} \frac{\partial y^\mu}{\partial \xi^a} \frac{\partial y^\nu}{\partial \xi^b} G_{\mu\nu} \right) \tag{6}$$

The calculation is done in Euclidean space and then Wick rotated to Minkowski space. Three remarkable results

1. Limiting our charges to flowing around a localised "current loop" produces Bio-Savart Law[5].
2. The functional average turns out to decouple the metric and field degrees of freedom so that the result is independent of metric and imposes no mass-shell condition. The calculation is thus carried out in a non-critical string theory.
3. The Wick rotation turns the statistical average into a time-ordered correlation function in finite temperature QFT; taking the large temperature limit, the properties of the Feynman propagator break the time-symmetry and naturally pick out the retarded solutions to Maxwell's equations.

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[5] Arχiv:1108.5094v2

Wilson Loops

Define a (non-local) operator - the Wilson Line[6]:

$$W\left(\mathbf{x}, \mathbf{x}'\right) = Tr\left[\mathscr{P}\left(\exp i \int_C A^\mu dx_\mu\right)\right] \tag{7}$$

for some path, C, joining the points $\mathbf{x}$ and $\mathbf{x}'$.

The (gauge invariant) *Wilson Loop* is this operator evaluated along a closed path through a chosen point. Stokes' theorem allows this to be writen in terms of $F_{\mu\nu}$, which with our ansatz suggests

$$\langle W \rangle = \int \mathscr{D}\mathbf{X} \exp\left(-S_{NG}\left[\mathbf{X}\right] - \frac{e^2}{2}\int_\Sigma \int_\Sigma d\Sigma^{\mu\nu}(\mathbf{x})\, \delta\left(\mathbf{x} - \mathbf{x}'\right) d\Sigma_{\mu\nu}(\mathbf{x}')\right) \tag{8}$$

[6]Phys Rev D **10**

Wilson Loops and QED

Gauge invariant operators (and observables) can be made up out of appropriate combinations of Wilson Loops.

- Strassler[7] showed how heavy quarks can produce the Wilson loops in Maxwell theory
- Quantum corrections arise from fluctuations of the world-lines of the quarks

In our work the interaction can be expanded in power series.

$$\langle W \rangle = \int \mathscr{D}\mathbf{X}\; e^{-S_{NG}} \Big(1 - \frac{e^2}{2} \iint d\Sigma^{\mu\nu} \delta\left(\mathbf{x} - \mathbf{x}'\right) d\Sigma_{\mu\nu} + \frac{e^4}{8} \iint d\Sigma^{\mu\nu} \delta\left(\mathbf{x} - \mathbf{x}'\right) d\Sigma_{\mu\nu} \iint d\Sigma^{\mu\nu} \delta\left(\mathbf{x}'' - \mathbf{x}'''\right) d\Sigma_{\mu\nu} + \dots \Big) \tag{9}$$

[7] Arχiv:9205205v1

Results

Theory is still a work in progress and there are still calculations to be completed. Calculation of first few terms of expansion show general features of the expansion which needs to be proven explicitly. In particular

1. First non-trivial term in expansion provides expected propagator

$$\oint\oint \frac{d\mathbf{x} \cdot d\mathbf{x}'}{\left|\mathbf{x} - \mathbf{x}'\right|^{D-2}} \tag{10}$$

2. Second term factorises into products of propagators
3. Anomalous metric dependence protected by factorisation and divergence of coincident Green's function
4. Regularisation plus normalisation able to counter divergent terms associated with self-interaction and vacuum energy

Conclusion

Current state of affairs:

1. A novel model of electromagnetism and QED is being investigated
2. Current calculations show fair agreement with accepted results
3. Calculation can be done in non-critical dimension with slightly irregular vertex operators inserted on the world-sheet due to decoupling of metric degrees of freedom

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Current state of affairs:

1. A novel model of electromagnetism and QED is being investigated
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Further work:

- Confirm the general structure of factorisation and regularisation
- Allow for dynamics of the heavy quarks - allow fluctuation of the boundary of the worldsheet
- Consider higher genus worldsheets to generate loop diagrams
- Comparison with QED - a test of the model
- Generalisation to non-Abelian gauge groups